# Angle of repose for two-dimensional particle aggregates from particle-size and -shape

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ABSTRACT: We investigate the dependence of the angle of repose on inter-particle friction and particles shape for heaps of dry granular materials built from polygons which flow out of a hopper. For any elongation, the angle of repose depends crucially on the coefficient of (dry/Coulomb) friction. For non-elongated particles, the angle of repose also depends crucially on the number of corners of the particles.

## 1 INTRODUCTION

In this article we investigate the influence of different parameters on the angle of repose. In both experiments and simulations via the discrete element method (DEM), the angle of repose, a fundamental quantity for the experimental classification of granular materials, depends crucially on the particles properties (**?**). Previous simulations for the angle of repose of dry granular materials used compositions of round particles and neglected friction (Gallas & Sokolowski (1993); Pöschel, T. & Buchholtz.(1993)) and relied on the interlocking of grain surfaces (see Fig.1) or neglected the rotational degree of freedom (Lee & Herrmann (1993)). As a result, the slopes of the heaps were not smooth and straight, but rough and curved, which is typical for wet/cohesive, not for dry granular materials (Barabasi & al. (1999); Schinner & Mattutis (1999)). These simulations resorted to using rough grounds to retain a certain verisimilitude, though heaps of dry sand can be built even on "microscopically" flat mirrors.

## 2 SETUP OF THE SIMULATION

We simulate sand heaps in two dimensions as an aggregate of polygons. with three degrees of freedom, two for the linear motion, one for the rotation. The

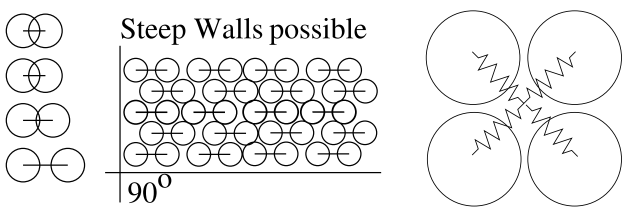


Figure 1. Dumbbell-model by Gallas (left) for different elongations, resulting in the possibility of a stacking with $90^o$ slope (middle) and clover-leaf-model by Pöschel (right).

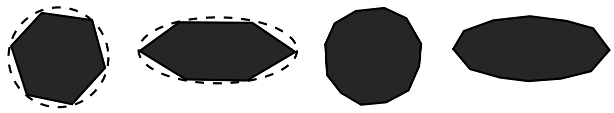

Figure 2. Construction of polygons from ellipses of various elongations, non-elongated and elongated particle with 6 corners, non-elongated and elongated particles with twelve corners (from left to right).

particles (about 4900) are "soft", so that the force between neighboring particles is computed proportional to their overlap area and their Young modulus ($10^7$ N/m for all simulations). Further components of the inter-particle-forces are a coefficient of restitution $\gamma < 1$ and the Coulomb friction coefficient $\mu$. The forces are used for the integration of Newton's equations of motion, the integration is performed by Gear Predictor-Corrector / Backward Difference Formula (for details, see Ref. (Matuttis (1998))). We limit our investigation to convex polygons which are inscribed into ellipses with varying length, and into circles for non-elongated particles (see Fig. 2). Though the simulation of concave polygons poses no problem in principle, concave surfaces increase the number of parameters in the model, and as initial investigation, we prefer the rather overseeable parameters of friction, elongation and number of corners. In the following, we will speak of non-elongated particles if

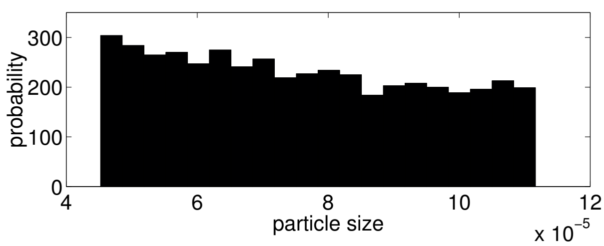


Figure 3. Typical size distribution used in the simulation; for the particle size, the area was given in units of $m^2$.

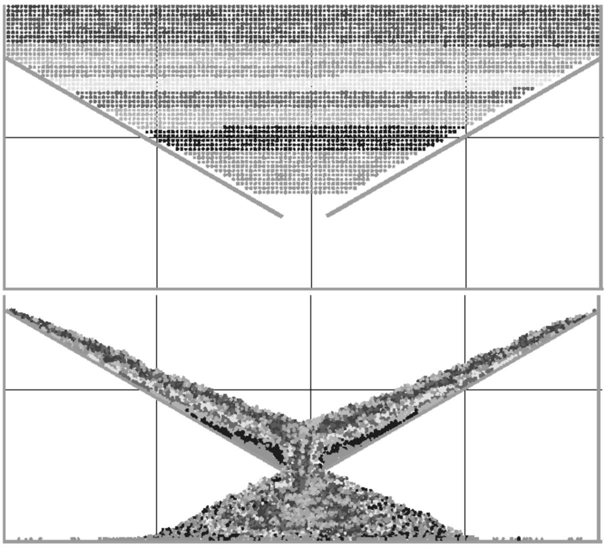

Figure 4. Initial state (top) and end configuration (bottom) of a simulation for the computation of the outflow from a hopper on a non-rough ground, $\mu = 0.6$. The slopes are straight, as in experiments with non spherical particles. Particles in neighboring layers have the same gray-scale, mixing of shades in the end configuration indicates mixing of layers.

the polygons were inscribed into circles. We will call the ratio between the longer and the shorter half axes of the ellipses into which the particles have been inscribed their ”elongation” for the sake of brevity. This allows to vary the corner number/ ”roughness” of the particles: More corners result in a smoother rounding of the particle outline, see Fig. 2. The corners for polygons with $n$ sides are chosen by increments of $360^o/n \pm 10\%$. We simulate a polydisperse system, with a fixed size dispersion (see Fig. 3 for a typical sample), because with mono-disperse size distributions, the particles can order locally in crystal structures. In reality, this behavior is absent in most real granular materials, except e.g. mono-disperse glass beads, where the particles show crystalline ordering near the surfaces of the system. Sliding along the crystal axis reduces the stability of the heap, so we use a polydisperse particle ensemble, which circumvents this artifact. The particles are initialized above a hopper geometry (see Fig. 4) and fall down at the beginning of the simulation. During the simulation, they aggregate on the hopper surface and slide down the slopes of the hopper. The walls which form the boundary and the hopper have for simplicity the same simulation parameters as the particles, i.e. the same Young modulus, coefficient of restitution and friction coefficient.

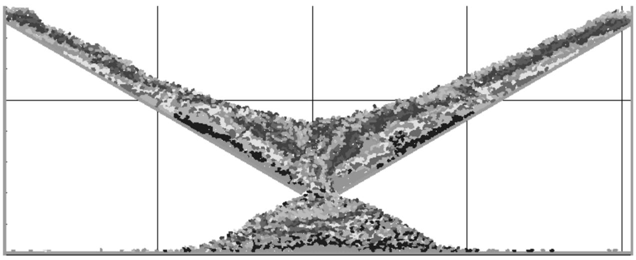

Figure 5. End configuration of a simulation of particles with five corners, smooth ground, $\mu = 0.6$.

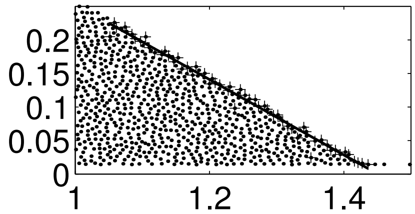


Figure 6. Center of mass of particles in the bulk of a right slope (dots), on the surface of the slope (+) and least squares fit to the data (solid line) for the determination of the angle of repose.

## 2.1 *Computation of the angle of repose*

Because the slopes of the heaps in our simulations are straight, except sometimes at the foot and near the top of the heap (see Fig. 5), the angle of repose can be obtained by a least-squares fit (see Fig. 6) to the center of mass of the highest particles in each segment of $\sim 0.5$ particle diameters length. Asymmetries in the heap shape are due to the intrinsic granular disorder. Initial particle configurations were computed with different random number sequences, i.e. the shapes of the particles, but not the average size distributions, were different for each run. The left and right slope of each heap were treated as independent configurations.

# 3 COEFFICIENT OF FRICTION

In granular materials, particles can either move by rolling or sliding. If the coefficient of friction is over a certain threshold, the particles prefer to move by rolling, if the particles are too rough or too elongated, they do not role, but slide. Whereas for small friction coefficients, also sliding on the ground can occur, for large coefficients of friction the buildup of the heaps takes only place via avalanches on the heap surface. For non-elongated polygons with five corners, we computed the dependence of the angle of repose on the friction coefficient (see Fig. 7, left). The average angle of repose increased linearly between $\mu = 0.2$ and $\mu = 0.4$ and saturated beyond $\mu \geq 0.5$. The reason for the saturation is, that the particle rotation and friction ”compete” as causes for the motion of the particles: For vanishing friction coefficients, the angle of repose can be extrapolated to 0 within error bars, consistent with the truism that materials with vanishing Coulomb friction (fluids) don’t form heaps.

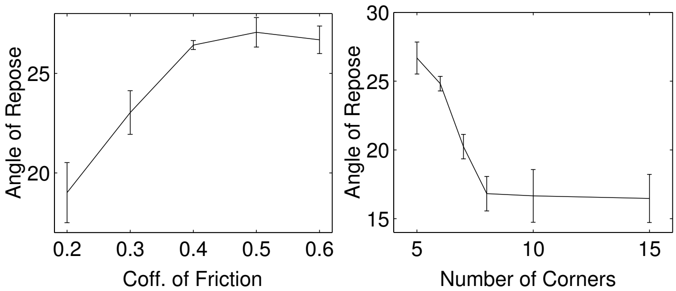


Figure 7. Dependence of the angle of repose on the coefficient of friction for nearly-regular pentagons (left) and dependence of the angle of repose on the number of corners of non-elongated nearly regular polygons (right). Averages and error-bars calculated for at least 6 slopes /independent configurations, $\mu = 0.6$.

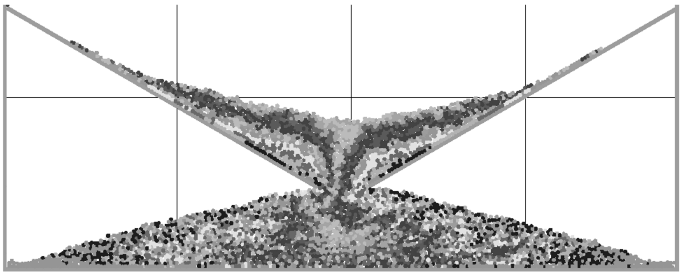

Figure 8. Final configuration of a simulation with non-elongated polygons with 25 corners, $\mu = 0.6$.

## 4 PARTICLE SHAPE

Because the literature of granular matter abounds with simulations of round particles, in this section we want to investigate the effect of a deviation from the round shape in the absence of a particle elongation, which is a kind of “macroscopic” roughness of the particles. Sphericity or its lack is crucial in the formation of the angle of repose: Whereas for polygons with five sides/corners any rolling can be thought to be essentially suppressed, especially on the ground, for the ”nearly round” particles, rolling is very likely.

### 4.1 *Number of corners and the Angle of Repose*

In this section, we explore the effect of number of corners on the angle of repose for non-elongated particles (nearly-regular polygons inscribed into a circle). The friction coefficient was set to $\mu = 0.6$, well in the regime where no change of the angle of repose due to the friction coefficient was observed any more (Fig. 7). From five to eight corners, the angle of repose decays practically linearly, and for larger corner numbers stays essentially constant. For 10 and 15 corners the outer particles already touch the walls, so that the data are unreliable. The remarkably small angle of repose for a heap made of polygons with 25 corners, nearly round particles, can be seen in Fig. 8.

### 4.2 *Effects for particle elongation*

For finite elongations, the angle of repose increases for particles with many corners because the tendency to roll is suppressed in comparison to non-elongated particles. For elongation 1.2 (snapshot of particles in Fig. 9, left) the angle of repose decays linearly from 5 to 10 corners, and from 10 to 15 corners decays hardly any more (Fig. 10). The angle of repose is higher than for non-elongated particles with the same number of corners. In contrast to elongations of 1.2 and smaller, for larger elongations (e.g. 1.4, Fig. 9, right)), there

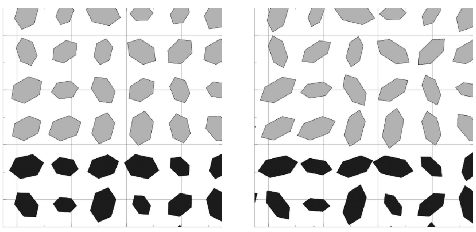

Figure 9. Zoom into the initial configuration for elongated hexagons with elongation 1.2 (left) and elongation 1.4 (right).

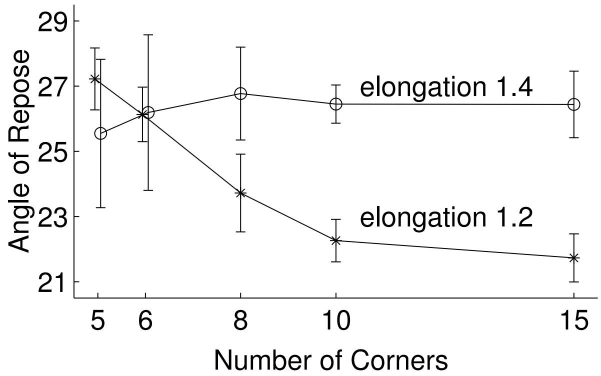


Figure 10. Dependence of the angle of repose on the number of corners for polygons with elongation 1.2 (stars), and for elongation 1.4 (circles), averages and error-bars calculated for at least 6 slopes /independent configurations, $\mu = 0.6$.

is no significant dependence of the angle of repose on the number of corners any more (elongation 1.4 in Fig. 10, circles). That means that for the elongation 1.4, the rolling is already essentially suppressed, the sliding dominates and the particle roughness does not play a crucial role. The error bars in Fig. 10 can be considered as a measure of the granular fluctuations in the formation of the angle of repose in two dimensions, which are more pronounced than can be expected in the three-dimensional case.

### 4.3 *Effect of a rough floor*

We tried to speed up the simulation by putting particles in a layer on the ground to increase the energy dissipation, which did not work out well. In the formation of the heap on the lower layer, both sliding of the damping layer and avalanching of the particles coming from above happened simultaneously. As can be seen in Fig. 11, the slopes are not straight and the angle of repose is not well defined: The particles from the damping layer are shoved into the wings with small angle of repose on the sides, whereas the particles which came through the hopper form the steep angle of repose via avalanching. As the damping layer constitutes a rough ground, this indicates how rough

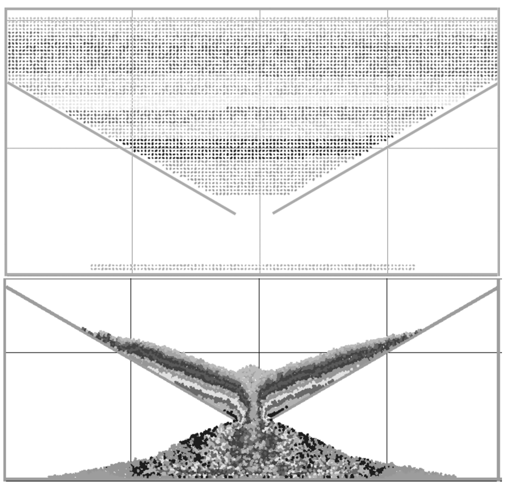

Figure 11. Initial state (above) and end configuration (below of a simulation for the computation of the outflow from a hopper on a rough ground, $\mu = 0.6$ onto a layer with particles which are finally shoved into the wings of the heap.

bottoms in simulations influence the angle of repose. In Fig. 12, we used elongated particles with a very small coefficient of friction $\mu = 0.2$. The particles touch the walls and form a nearly horizontal layers. Only on this rough layer, a heap forms in the middle under the hopper with a measurable angle of repose, due to the rough granular layer below, not due the particle interactions with the ground. This shows how the angle of repose is affected by rough grounds.

## 5 CONCLUSION

We have discussed the effect of the shape of particles on the angle of repose, a paradigm on the competition between rolling and sliding in granular materials. We have shown that the angle of repose for dry granular materials depends crucially on the particles shape and the Coulomb friction coefficient. For non- or moderately elongated particles (i.e. up to an elongation 1.2), we found a strong dependence of the angle of repose on the particle roughness, i.e. for our polygonal particles the angle of reposed decreased with increasing number of corners. This result is in marked contrast to the results for triaxial compression, where one finds hardly any dependence of the stress-strain-curve on the particle roughness (Matuttis & al. (2003)). For larger elongations, the effect of the particle roughness on the angle of repose is suppressed, because no rolling takes place anyway, and the angle of repose depends mostly on the friction coefficient. Simulations which neglect any of these influences will not be able to reproduce neither the statics (angle of repose, stress distribution) nor the dynamics (e.g. avalanches, force networks) for realistic materials. The whole mechanism of formation of heaps and angle of repose is the result of the particle properties (shape and inter-particle friction) on the one hand, and of sliding on the ground and avalanches on the other hand. For static granular systems with free surfaces, the shape is a parameter like e.g. friction, for non-elongated particles, even the deviation from the circular shape has to be included in the modelization, so unsatisfying this may be from the standpoint of "rational" (i.e. parameter-free) mechanics.
Direct comparisons of two-dimensional simulations, where the pressure on the ground results from the total mass of the particles, with experiments are difficult, as systems between two walls are very narrow silos, where due to the Jansen-effect: the friction with the walls reduces the pressure on the ground and the internal shear stresses in the granular bulk. For regular mono-disperse flat pentagons, the experimental angle of repose found in a rotating drum (Cantelaube). The friction coefficient was not recorded in (Cantelaube), but due to the used plastic materials can be assumed to be in our "saturated region" beyond $\mu = 0.4$, and the Jansen effect probably had no influence due to the relative width of the drum and the particles. The experimental angle was larger (32 degrees) than our simulated angle of polygonal pentagons (Fig. 7), probably because the horizontal forces in drum were partly compensated by the curved bottom in normal direction, not only by the friction with the plane as in our simulations. A further possible stabilization in the experiment occurred by the frequent side-by-side contacts of the mono-disperse pentagons, which were rarer in our polydisperse simulation. Where both the experiment and our simulation agree is the wide, nearly Gaussian distribution of the critical angle (half-width in the experiment is nearly $\pm 10$ %), which is comparable to our simulations of non-elongated particles in the saturated regime. That this scattering is relatively larger than for the three-dimensional case may be attributed to the fact that two straight lines in two dimensions "nearly always" intersect, so that strong forces meet and some point in the heap and alter the structure by shear.

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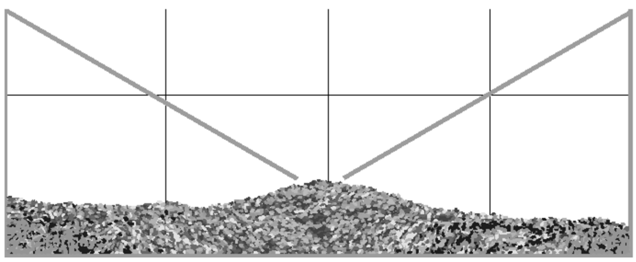

Figure 12. Final configuration for a rough ground ($\mu = 0.2$, elongation 1.4, five corners): Below the opening of the hopper, a heap with straight slopes is still recognizable.