# History Dependence Of The Density Distribution On Granular Heaps

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**Abstract.** In previous computational work by one of the authors, the pressure minimum under heaps was shown to result from the construction history. In this work we will show that the construction history is actually embedded in the average density of the granular material.

**Keywords:** History dependence, pressure dip, granular density, density measurement
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## INTRODUCTION

In the second half of the 1990s, a massive controversy arose in the granular community about the occurrence of "pressure dips" in granular heaps, relative minima in the pressure distributions in the middle of heaps between larger pressure amplitudes. The experimental results and the opinions were pretty much split between the physics- and powder mechanics- community on the one hand, who favored dips, while researchers close to the geotechnical community were rather sceptical, see [1] as well as the references in [2, 3] for an overview. Arching as a "prime suspect" for the formation of the pressure dip was already mentioned in the experimental papers on the pressure dip [4]. Arching has been discussed in granular material research for more than hundred years: The first references for "Bogenbildung" (arching) in Terzaghi's textbook on "Erdbaumechanik" (geotechnics) [5] date from the last quarter of the nineteenth century [6, 7]. One of the authors realized that the contradicting experimental results occurred often for different approaches of building heaps, and "pressure dips" occurred not in layered (Fig. 1), but only in wedge sequences (Fig. 2). The proposition that different construction histories (or processes) led to different pressure distributions, based on two-dimensional simulations of polygonal particles [2], was later confirmed experimentally in three dimensions by another group [8]. The question was, whether arching was possible in homogeneous media (due to e.g. inhomogeneous mobilization of static friction) or due to a material inhomogeneity. "Generic" models with assumed pressure redistributions below are no help: Depending on the choice of geometry, different pressure distributions can be obtained, no matter whether the model is formulated in terms of volume elements of finite (Fig.3) or infinitesimal length scale (partial differential equations, for a discussion of the correspondence between different models see Schinner et al. [3]). After in heaps with different building histories different density distributions were discovered [9], we looked for a method which could be used for an experimental verification without undue experimental or financial effort. The first experimental results are presented here, as well as simulation results in two dimensions to verify the consistency.

## EXPERIMENT

For a long time, we had been looking for a feasible measurement method, cheaper (and maybe more accurate) than NMR [10] and less dangerous than X-rays.Finally,

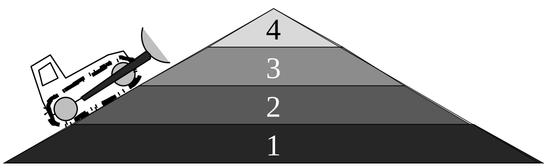


**FIGURE 1.** Layered sequence, the construction method dominant in the civil engineering literature.

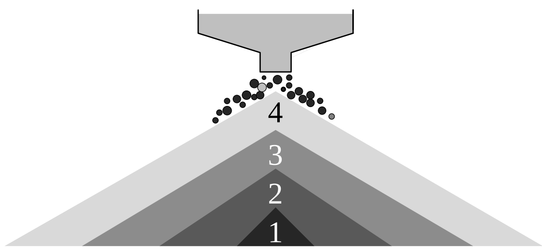


**FIGURE 2.** Wedge sequence, the construction method dominant in the powder engineering literature.

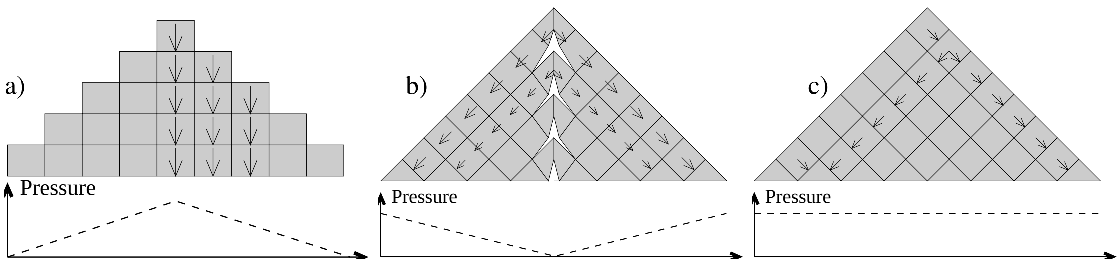


**FIGURE 3.** "Generic" models for the pressure distribution under a heap with maximum in the middle (a), minimum in the middle (b) and constant pressure (c).

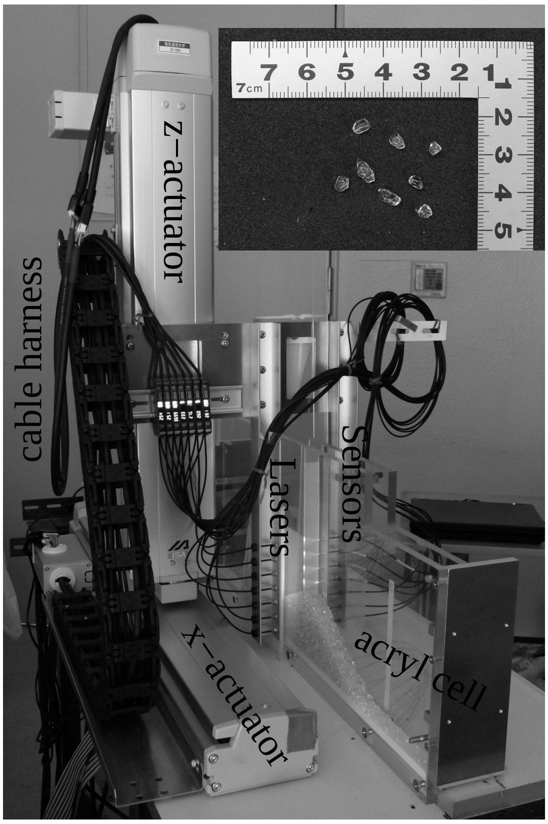


**FIGURE 4.** Experimental setup: Only the lowest laser sensor was used. Inset shows beads used for the experiment.

we[11] succeeded in calibrating Laser-sensors (Keyence Fiber Optic Sensor /FS-V21RM, FU-77, MS-H50with Keyence Data Acquisition/Monitoring System /NR-110) for density measurements of densely packed glass beads: The intensity loss (due to dispersion) in glass beads decreases exponentially with the layer depth, which is not a matter of course, as foam behaves differently, see Durian [12]. This kind of calibration to resolve density variations of few percent [11] takes considerably more accuracy and experimental skill than the detection of moving plugs (and the empty space between) in other experiments [13]. Another issue was the building of the heap: We wanted to be able to direct a point-source under reproduceable conditions at arbitrary positions. We settled for a "linear robot system" with computer-controlled positioning for the x- and z-axis, IAI (Intelligent Actuator Inc) / ICSA2-ZICM-A-60-40B-T1-5L-CT, see Fig. 4. Because the measuring depth of the system is limited by the laster intensity, we built a heap between Acryl-plates of 1 cm thickness. Such systems are sometimes called "two-dimensional", but the difference to two-dimensional simulations is that not the whole mass of the heap is actually carried by the bottom, but a significant portion of the weight is carried by the walls (Jansen-Effect). Both the filling and the measurement was performed with the robot, the filling by moving a hopper, the measurement by moving the laser-sensors. We used non-spherical glass-grains of about 2 to 4 mm linear size of irregular shape, see insert in Fig. 4. In the vertical direction, the density scans were performed in a distance of about the particle size, whereas in the horizontal directions, the measurements are practically continuous due to the high sampling frequency of the laser sensors. The measurements in the lowest layers show shading differences in shorter length-scales than in the upper part of the heap. This is probably a compaction effect close to the ground, as we made every effort to exclude reflexions etc.. The wedge sequence shows clearly a high-density core (Fig. 5, above), while in the heap with the layered sequence the density fluctuations are distributed over the whole heap (Fig. 5, below). Different legends have been used because the maximal and minimal densities are different, in the same shading, the contrast vanishes.

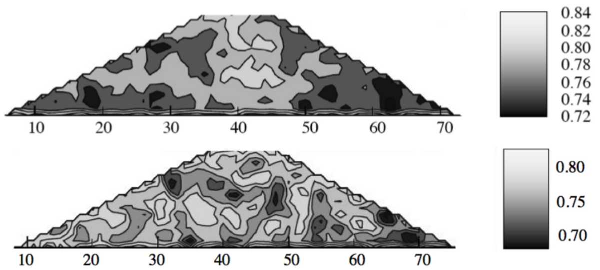


**FIGURE 5.** Experimental density for the wedge sequence (above) and the layered sequence (below).

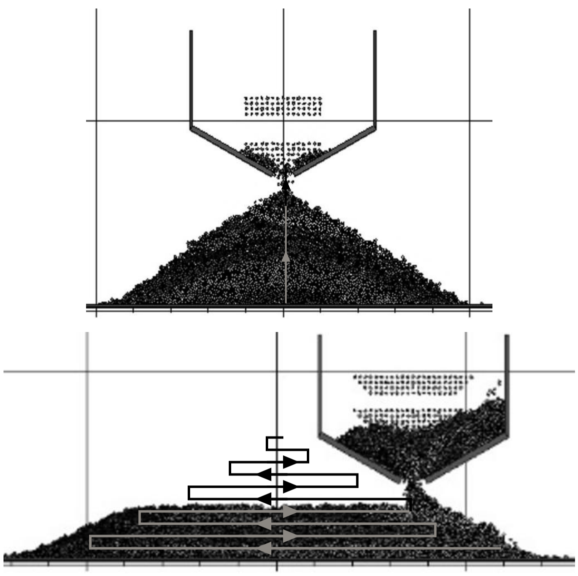

**FIGURE 6.** Trajectory for the hopper in the wedge sequence (above) and layered sequence (below), trajectory up to the snapshots in gray, later trajectory in black, superposed on snapshots from the simulation. Above the hopper, the batches for the deposition can be seen.

## SIMULATION

We carried out two-dimensional simulations for heaps of polygonal particles built in wedge- and layered sequence. In the force law, the Young modulus $Y$ and the overlap area $A$ between particles is used as a measure of the elastic contact force and the time derivate $dA/dt$ as damping term in normal direction. Additional, static and dynamic friction ($\mu = 0.6$) after Cundall and Strack [14] was implemented. For the time integration, the (implicit and stiffly stable) Gear-Predictor-Corrector (Backward-Difference Formula) of fifth order ("BDF5) was used. Detailed description, as well as the limits of linear, Hertzian- and Wedge-contacts which are reproduced by the force law can be found in Shourbagy et al. [15]. We used elongated pentagons with Young modulus $Y = 1 \cdot 10^7$ N/m and density $\rho = 2790$ kg/m$^2$. which were generated by inscribing them into ellipses with half-axes in a ratio of 10:7 with a nearly linear distribution of cross-section areas of 1.1mm$^2 \pm$ 0.4mm$^2$. The particles were added batch by batch into the hopper (see Fig. 6). The settings were the same for two building methods except for the 0 position and the movement of the hopper. In the wedge sequence, the hopper was kept in the center of the system and lifted in y-direction. In the layered sequence, the hopper was moved to the left and right along the x-axis and lifted up along in y-direction (ten layers in total) to generate layers and finally form a heap, see Fig. 6. The $x - z$ dimensions of the heap where approximately those of the simulation. Computationally, we have several possibilities to perform the homogenization, i.e. to define the densities of the particles in the continuous volumes, see Fig. 7. If we want to define the ratio of the area of the polygons $V_i$ included in the grid and the area of the grid $V_g$ used for the homogenized density

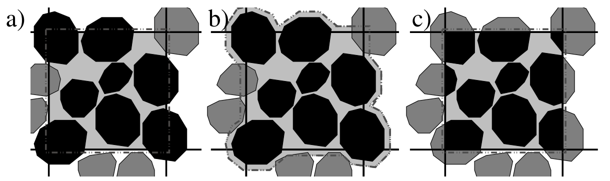


**FIGURE 7.** The three homogenization methods for computing the density distribution: In black the grains volumes which are counted, while the dotted line indicates the volume $V_g$ used at the given square.

$$\rho = \frac{V_1 + V_2 + \cdots + V_n}{V_g},$$

we have three obvious possible definitions for the $V_i$. The computationally simplest way is to choose the volumes of all the particles which have their center of mass in $V_g$ (Fig. 7 a)), which is the least accurate method [16]. Adding the volume of the polygons which extend beyond the grid $V_g$ to the grid volume $V_g$ (Fig. 7 b)) or taking only the volume inside the grid $V_g$ (Fig. 7 c)) gives accurate, equivalent methods [16], of which we choose the latter. The results of the simulation in Fig. 8 are consistent with the results of the experiment in Fig. 5: In the wedge sequence, we have a high-density core flanked by sides of lower density. For the layered sequence, we have more or less homogeneous density fluctuations throughout the heap, with lower densities on the upper layers. Both in the experiment and the simulation, the density varies between 70 % and 80 % of the bulk density.

That the density can vary below and above 5 % of the closest packing for mono-disperse spheres (74.0. . . %) is due to the fact that our particles are neither spheres

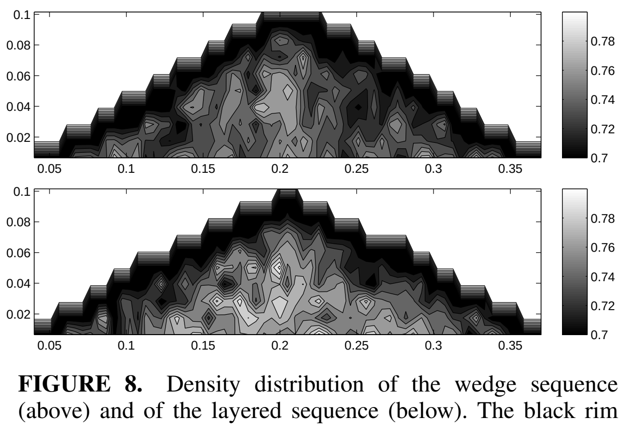


**FIGURE 8.** Density distribution of the wedge sequence (above) and of the layered sequence (below). The black rim of low density is due to the data processing, where the upper, rough layer of grains produce reduced densities.

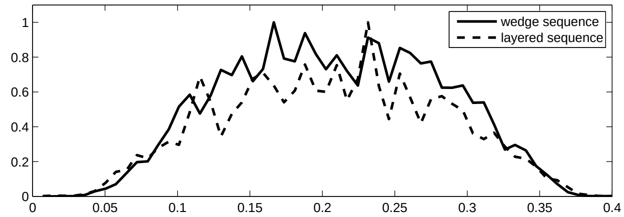


**FIGURE 9.** The bottom pressure distributions of the wedge sequence and the layered sequence. The maximal pressure is set to 1. Lines are drawn to guide the eye.

nor are they mono-disperse. Experimentally, we were not able to devise a low-cost pressure-measurement method, we could only compute the pressure distribution for our simulation. The y-axis component of the force between the particles and the ground was summed per unit length, so the pressure is

$$\sigma = \frac{f_1 + f_2 + \cdots + f_n}{L_g},$$

with moving averages, i.e. overlapping line elements were taken in Fig. 9. The plot of the bottom pressure distribution for the wedge sequence (solid line in Fig. 9) is consistent with a pressure minimum near the center of the heap (though the fluctuations are large), while for the layered sequence (dashed line in Fig. 9) the fluctuations are larger than any possible minimum. (Conclusive data were obtained by Schinner et al. [3] with averages over 10 heaps).

## CONCLUSIONS

From our simulations and experiments, we can conclude that the "memories in sand", as they have been called so poetically by Vanel et al. [8], are embedded via the density. Structure might have similar effects, but we found (in our initialization with disorder) neither experimental evidence not references for that. It is rather counterintuitive that the pressure-minimum in the middle occurs due to a high-density core above it for heaps built in a wedge sequence. The quasi-two-dimensional experiments and the two-dimensional simulation give consistent density variations. While we think that our experimental method could be improved in resolution, nevertheless the density inhomogeneities can be safely identified as the cause of the history dependence. Density inhomogeneities have other far-reaching effects, e.g. on the avalanching from sand-heaps, where they seem to ruin in the experiment the power-law distribution of the avalanche sizes [17] which are predicted by self-organized-criticality-theories. While recent continuum theories have taken into account effects of the construction history [18], which is an advance compared to the nearly "hydrodynamic" theories with conjectured stress distributions from the 1990s, the density is still treated as an auxiliary variable (and not given in Ref. [18], only the stresses are). We hope that we have made a strong case that for realistic theories, both the density and the stresses have to be taken into account.

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