THE EFFECT OF PARTICLE SHAPE AND FRICTION ON THE STRESSES IN HEAPS OF GRANULAR MEDIA

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We simulate static model sandpiles made of polygonal particles in two dimensions (2D) and focus on the stresses inside the pile. The shape of the simulated heaps depends on the construction history, i.e. whether the heap was initially ordered or randomly created and whether the pile was put upon a flat or rough surface. On the other hand, there is only a weak dependence of the stresses in the pile on the interaction parameters.

1 Introduction

The stress distribution in static granular media has been examined with different methods and approaches.[1,2,3] The unintuitive outcome of an experiment [4] on the stress distribution under a heap of granular material (fertilizer/sand) has challenged the theoretical community, as the vertical stress at the bottom has a relative minimum in the middle of the heap. In contrast, a theoretical prediction [5] for particles arranged in a two-dimensional heap with 60° slope gives constant vertical stress on the whole width of the heap. This model took into account infinitely "hard" particles with no friction only on a hexagonal lattice and open horizontal contacts. Another approach using molecular dynamics (MD) modeling of smooth "soft" spheres may lead to a dip unter certain boundary conditions [6] and displays the eminent importance of both the fluctuations and the contact network for the stress distribution.

Our aim in this paper is to include more physical features like friction and non-spherical shapes of the particles in the model by computational means and to investigate the effect of different construction methods and boundary conditions.

2 Algorithm

The simulation is performed using a molecular dynamics algorithm, where Newton's equations of motion are integrated numerically using a predictor corrector scheme of fifth order. [7] The model for the interaction of two polygonal particles will be outlined shortly.
Two particles with index i and j and center of mass C_i and C_j are shown schematically in Fig. 1. If the particles overlap, we assume the repulsive force as proportional to the area of this overlap. For convex polygons, the shape of the overlap is again a convex polygon (see the zoom at the right of Fig. 1). The force acts normal to the line connecting the intersection points of the outline of the two polygons. The forcepoint S_{ij} is given as the center of mass of the overlap polygon. As the direction of the force is not necessarily parallel to the connection $\vec{r}_i$ between the centers of mass C_i and the forcepoint S_{ij}, this leads to torques even if no tangential friction

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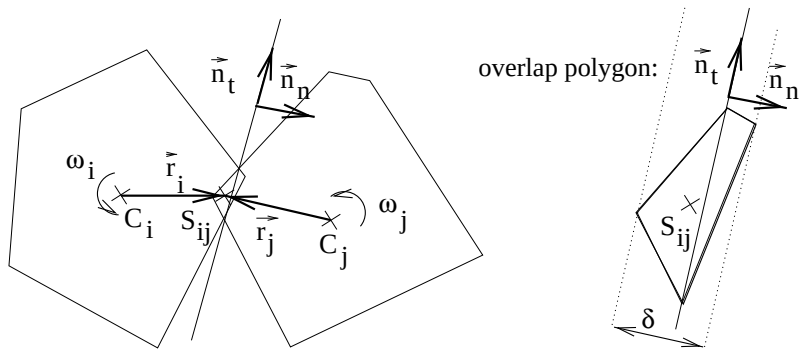

Figure 1: Schematic drawing of the interaction geometry for two colliding polygonal particles.

is present. The relative velocity

$$\vec{v}_r = \vec{v}_i - \vec{v}_j + (\vec{\omega}_i \times \vec{r}_i - \vec{\omega}_j \times \vec{r}_j)$$

at the force point S_{ij} depends on the linear velocities v_i, v_j, the rotational velocities ω_i, ω_j and the distance between the force point S_{ij} and the center of mass of both particles, C_i and C_j. The relative velocity can be decomposed into its normal and tangential part (see also Fig 1):

$$v_{r,n} = \vec{v}_r \cdot \vec{n}_n, \quad \text{and} \quad v_{r,t} = \vec{v}_r \cdot \vec{n}_t.$$

2.1 Normal force

For two polygonal particles, we assume the normal force due to the elastic deformation as proportional to area A of the overlap polygon as[8]

$$|F_e| = YA/r_{ij}, \quad r_{ij} = 2(|r_i| + |r_j|)$$

where Y is the Young modulus and r_{ij} is a characteristic length. The strength of this definition is the dependence of the force and therefore of the collision time t_c on the shape of the particle contact. For normal collisions of a rectangular particle with a flat surface a forcelaw linear in the overlap δ results, which causes a constant contact duration for free collisions. For rounded contacts one obtains $|F_e| \propto \delta^{3/2}$ and therefore a decay of the collision time with the collision velocity of $t_c \propto v^{-1/5}$. This corresponds to the analytic result for the Hertzian contact law for round particles.[9,10] For contacts between edges and flat surfaces, we have $F \propto \delta^2$ and $t_c = v^{-1/3}$. This type of contact is unstable, even for small rotations, and we observe this type of contact rather seldom in the bulk.
The elastic deformation of the particles is assumed to lead to a "viscous" damping force (related to the coefficient of restitution) which is taken as

$$F_v = -\gamma_n \sqrt{Ym}\; v_{r,n} \delta^{\alpha},$$

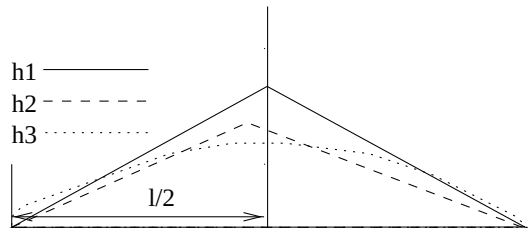

Figure 2: Shape of the heaps h1: heap on a hexagonal lattice; h2: heap on a rough surface; h3: heap on a smooth surface

with the reduced mass $m = (m_i m_j)/(m_i + m_j)$ and a phenomenologial dimensionless damping constant γ_n. F_v is in general a complex nonlinear function of $v_{r,n}$ and the penetration depth δ or equivalently the area A. The dissipation can be estimated [9] and one obtains a velocity-dependent restitution coefficient. As we are interested in static situations with vanishing velocities, and because the majority of the contacs is of the type where $|F_e| \propto \delta$, we chose the simplest interaction law with $\alpha = 0$ for F_v. The normal force is given as $F_n = F_e + F_v$. We used $Y = 10^7$ N/m and $\gamma = 0.05$ or 0.1, $g = 9.81$ m/s^2 and particles of about 1mm diameter with density ρ =5000 kg/m^3.

2.2 Tangential force

For the tangential direction, we adapt the force law proposed by Cundall and Strack [11,12], i.e. during the contact a tangential spring ist attached. Therefore, we obtain a tangential force which is able to stabilize heaps for polygonal particles with a rotational degree of freedom even on smooth surfaces. The change per time step of the tangential force is proportional to the relative displacement $v_{r,t}\mathrm{d}t$ of the contact point per time step and the scaled Young modulus $k_t Y$. The total tangential force is limited by Coulomb's law of friction so that

$$F_t = \min\left(\mu F_n, \sum_{t'} \Delta F_{t'}\right), \quad \text{with} \quad \Delta F_t = -k_t Y v_{r,t}\mathrm{d}t,$$

where the sum has to be performed over all time steps during the contact. We used friction coefficients $\mu = 0.0, 0.3, 0.6$ and $k_t = 0.1$.

3 Effect of friction and shape

The geometry of polygonal particles and the friction can introduce torques on the particles in such a way that the stress tensor may be non-symmetric. Therefore, we average over several particles so that the stress tensor becomes approximately symmetric. For details on the stress calculation, see references.[6,15]
We simulate three different kinds of boundary conditions: In the first case, the particles are initially ordered on a hexagonal lattice (h1). In the second case, the particles are dropped onto a slightly rough surface (h2) and in the third case onto a smooth surface (h3). Due to the particles which impact onto the top of the second and third model heap, the angles of repose are smaller than in the case of the first

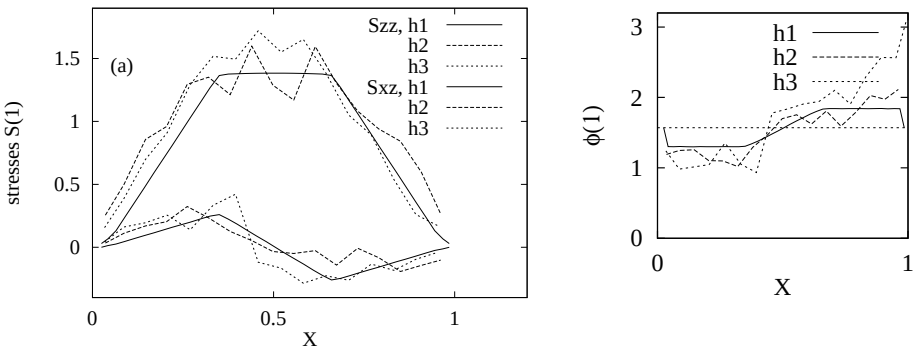

Figure 3: Stress distribution S_{zz} and S_{xz} for the different heaps (left) and angle ϕ about which the major principal axis is tilted from the horizontal in counterclockwise direction (right).

one, see Fig. 2. In the case of h3, the shape was not triangular due to the reordering of particles when the heap compactifies under its own weight.

3.1 Heap on a hexagonal grid

We simulated a heap of polygonal particles with 12 sides and with different friction coefficients and compared them to the simulations of round particles without friction.[15] The angle of repose of the heap was by construction 30°. The base of the heap was $L = 100$ particle diameters in the lowermost layer where the particles are fixed. The heap is built up layer by layer and is then relaxed until most of the energy is dissipated. The stress distribution for the heap on the hexagonal lattice for $\mu = 0.0, 0.3, 0.6$ did not change noticeable with the friction coefficient μ. In Fig. 3, the stress for heap $h1$ is given only for $\mu = 0$. It is reasonable to suppose that on a hexagonal grid (due to the construction) all the stresses are promoted along the normal directions of the particle contacts. The solution for the stress curves becomes purely a result of the underlying lattice geometry.[13,14] Mounting the lowermost layer on springs (no figures shown) and permitting vertical motion did not change the qualitative outcome of the simulations either.

3.2 Heaps with random particle configuration

We use the above simulation to compare it to simulations of heaps which are set up by dropping particles from a "point source" onto the middle of the heap. The number of particles in the lowermost layer was again about 100, after all particles are dropped onto the heap. We continue the simulation until most of the kinetic energy is dissipated.

Due to disorder, fluctuations in the stresses can be seen. The slopes are slightly smaller than in the case of the polygonal heap, nevertheless the distribution of the stresses in z-direction is similar. No relative minimum in the normal stress in the center of the heap can be observed for the parameters used here. Also the angle ϕ for which the major principal axis is shifted counterclockwise from the horizontal

does not change dramatically for the different situations.

4 Conclusion

We present simulations of model-piles made of polygons in 2D. Along with simulations using spherical particles with different interaction laws [15], we can state that the results for a model sandpile as proposed in[5] do not change by simply introducing more physical features, like elasticity of the particles, nonlinearity of the interaction laws and a rotational degree of freedom, coupled by friction. Up to now, we cannot reproduce the outcome of the experiment in reference.[4]
As a next step, the effects of the construction of the heap have to be studied in more detail. Wittmer et al.[3] proposed a theory which gives a minimum of the vertical stress, if the heap is built up from a point source so that the configurations inside the pile are not disturbed. This does not apply for the simulations presented here. For heap h1, the whole structure is built up and relaxed simultaneously. For the heaps h2 and h3, where the heap is created by dropping particles, strong reordering due to the rather large impact velocity of the particles is induced inside the heap. The effect is most marked for the flat bottom (h3).
Currently we try to set up the heap in such a way, that the internal structure is not influenced too much by addition of new particles.

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References

1. F. Radjai, M. Jean, J. J. Moreau, S. Roux, Phys. *Rev. Lett.* **77**, 274 (1996)
2. C. h. Liu, S. R. Nagel, D. A. Schecter, S. N. Coppersmith, S. Majumdar, O. Narayan, and T. A. Witten, *Science* **269**, 513 (1995)
3. J. P. Wittmer, M.E. Cates, P. Claudin, *J. Phys.* **I**, (1996)
4. J. Smid, J. Novosad, in *Proc. of 1981 Powtech Conference, Ind. Chem. Eng. Symp.63,* D3/V/1-D3/V/12 (1981)
5. D. C. Hong, *Phys. Rev. E* **47**, 760 (1993)
6. S. Luding, submitted to *Phys. Rev. E.*
7. M. P. Allen, D. J. Tildesley, *Computer Simulation of Liquids*, Clarendon 1987
8. H.-J. Tillemans, *Molekulardynamik-Simulationen beliebig geformter Teilchen in zwei Dimensionen*, PhD thesis, KFA Jülich 1995 (in German)
9. S. Luding, E. Clément, A. Blumen, J. Rajchenbach, J. Duran, *Phys. Rev. E* **50**, 4113 (1994)
10. N. Brilliantov, F. Spahn, J. Hertzsch, T. Pöschel, *Phys. Rev. E* **53**, 5382 (1996)
11. P. A. Cundall, O. D. Strack, *Geotech.* **29**, 47 (1979)
12. J. Schäfer, S. Dippel, and D. Wolf, *J. Phys. I France* **6**, 5 (1996)

13. K. Liffman, D. Y. C. Chan, B. D. Hughes, *Powder Technology* **72**, 225-267 (1992)
14. K. Liffman, D. Chan, B. Hughes, *Powder Technology* **78**, 263 (1994)
15. S. Luding, H.-G. Matuttis, in *Friction, Arching and Contact Dynamics,* eds. D. Wolf and P. Grassberger, World Scientific, Singapore 1997